ItEs 2010

September 7-10, 2010
Badajoz (Spain)

BOOK OF ABSTRACTS

Seventh Italian-Spanish Conference on General Topology and its Applications

UNIVERSIDAD DE EXTREMADURA
Departamento de Matemáticas

Seventh Italian-Spanish Conference on General Topology and its Applications

Badajoz, September 7-10, 2010

BOOK OF ABSTRACTS

Departamento de Matemáticas, Universidad de Extremadura
06006 Badajoz - Spain

Contents:

Scientific Committee:

Giuseppe Di Maio (*Seconda Università di Napoli, Italy*)
Mª Isabel Garrido (*Universidad Complutense de Madrid, Spain*)
Roberto Moresco (*Università di Padova, Italy*)
Jesús Rodríguez López (*Universidad Politécnica de Valencia, Spain*)
Manuel Sanchís (*Universitat Jaume I, Spain*)
Gino Tironi (*Università di Trieste, Italy*)

Organizing Committee: (*Universidad de Extremadura, Spain*)

Teresa Arias-Marco
Adrián Gordillo
Francisco Montalvo (chairman)
M. Ángeles Mulero
Antonio A. Pulgarín
Batildo Requejo

Support: The Conference is supported by

Universidad de Extremadura, Departamento de Matemáticas
MICINN, Ministerio de Ciencia e Innovación
RET, Red Española de Topología
RSME, Real Sociedad Matemática Española
La Caja de Ahorros y Pensiones de Barcelona, "la Caixa"
Excmo. Ayuntamiento de Badajoz, Concejalía de Turismo

Introduction:

This book contains the abstracts of the talks that have been presented at the Seventh Italian-Spanish Conference on General Topology and its Applications (ItEs2010), held in Badajoz, Spain, from 7 to 10 of September, 2010. We apologize to the authors if, for get a unified appearance, there have been some changes in the abstracts.

Previous meetings have been held in Gandía (Spain) - 1997, Trieste (Italy) - 1999, La Manga del Mar Menor (Spain) - 2000, Bressanone (Italy) - 2001, Almería (Spain) - 2004, Bressanone (Italy) - 2007.

In this edition of the conference the number of registered participants was 53. The scientific program includes 4 main talks, 8 invited talks and 22 contributed talks.

Main speakers:

Angelo Bella (Università di Catania, Italy)
Anna Di Concilio (Università di Salerno, Italy)
Jesús A. Jaramillo (Universidad Complutense de Madrid, Spain)
Óscar Valero (Universitat de les Illes Balears, Spain)

Invited speaker:

Francisco Balibrea (Universidad de Murcia, Spain)
Gianni Bosi (Università di Trieste, Italy)
Félix Cabello (Universidad de Extremadura, Spain)
M. Ángeles de Prada (Universidad del País Vasco, Spain)
Jesús M. Domínguez (Universidad de Valladolid, Spain)
Salvador Hernández (Universitat Jaume I, Spain)
Miroslav Hušek (Charles University, Czech Republic)
Jorge Picado (Universidade de Coimbra, Portugal)

The Organizing Committee of ItEs2010 wants to thank all the participants, especially the main and invited speakers and the Scientific Committee, their contribution to the conference.

Schedule

ItEs 2010

Badajoz (Spain)

September 7 – 10, 2010

Schedule:

Lecture room: Classroom PhD, Faculty of Science, Juan Remón Camacho building

Registration & Check-In: Aula 10, Juan Remón Camacho building of the Faculty of Science

Monday September 6, 2010

18:00 – 20:00 Registration

20:30 – 21:30 Welcome cocktail

Registration will continue on Tuesday, 7

Tuesday September 7, 2010

Morning session

09:30 – 10:00 *Opening Ceremony*

10:10 – 11:00 ANNA DI CONCILIO (Università di Salerno, Italy)
Action, uniformity and proximity (page 23)

11:00 – 11:30 *Coffee break*

11:30 – 11:50 GUGU MOCHE (University of South Africa, South Africa)
Recent results on the smallest ideal of $\beta\mathbb{N}$ (page 63)

11:50 – 12:10 JESÚS RODRÍGUEZ-LÓPEZ (Universidad Politécnica de Valencia, Spain)
Miss topologies compatible with sequential upper kuratowski-painlevé convergence (page 67)

12:10 – 12:30 GINO TIRONI (Università di Trieste, Italy)
Pseudoradial order of pseudoradial spaces (page 71)

12:30 – 13:00 JESÚS M. DOMÍNGUEZ (Universidad de Valladolid, Spain)
The algebra of those continuous functions on X that are locally in a subalgebra of $C(X)$ (page 37)

Tuesday September 7, 2010

Afternoon session

16:00 – 16:30 María Angeles De Prada (Universidad del País Vasco-Euskal Herriko Unibertsitatea, Spain)
Domain-valued functions vs scales (page 36)

16:30 – 16:50 Athanasios Megaritis (University of Patras, Greece)
On dimension-like functions $dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$ and $Dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$ (page 62)

16:50 – 17:10 Dimitrios Georgiou (University of Patras, Greece)
On a new relative invariant covering dimension (page 54)

17:10 – 17:40 *Coffee break*

17:40 – 18:00 Juan-Luis García Zapata (Universidad de Extremadura, Spain)
The conditioning of the winding number of plane curves (page 52)

18:00 – 18:30 Francisco Balibrea (Universidad de Murcia, Spain)
On the periodic-recurrent property on dendrites and dendroids (page 31)

Wednesday September 8, 2010

9:30 – 17:00 Conference excursion. Guided tour of the city of Cáceres (World Heritage city)

Thursday September 9, 2010

Morning session

09:30 – 10:00	Gianni Bosi (University of Trieste, Italy) Topologies corresponding to continuous representability of preorders (page 32)
10:00 – 10:20	Esteban Induráin (Universidad Pública de Navarra, Spain) Topological spaces whose topology is generated by a binary relation (page 56)
10:20 – 10:40	Erdal Ekici (Canakkale Onsekiz Mart University, Turkey) On pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces (page 48)
10:40 – 11:00	Maria João Ferreira (University of Coimbra, Portugal) Insertion of real functions in completely normal point-free spaces (page 51)
11:00 – 11:30	*Coffee break*
11:30 – 11:50	Miguel Angel Sánchez-Granero (University of Almería, Spain) Fractal dimensions for fractal structures: a haussdorf dimension generalization (page 69)
11:50 – 12:10	Manuel Fernández (Universidad de Almería, Spain) Applications of fractal dimensions for fractal structures (page 49)
12:10 – 13:00	Oscar Valero (Universidad de las Islas Baleares, Spain) An overview of complexity spaces and its applications to computer science (page 27)

Thursday September 9, 2010

Afternoon session

16:00 – 16:30 Félix Cabello (Universidad de Extremadura, Spain)
Nonlinear banach-stone theorems for lattices of lipschitz functions (page 34)

16:30 – 16:50 Inderasan Naidoo (University of South Africa, South Africa)
On certain covering properties in structured frames (page 64)

16:50 – 17:10 Raja Mohammad Latif (King fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia)
On G.Λ_{π}^{gp}-sets, Λ_{π}^{gp}-closure operator and the associated topology $\tau^{\Lambda_{\pi}^{gp}}$ (page 59)

17:10 – 17:40 *Coffee break*

17:40 – 18:00 Dikran Dikranjan (Udine University, Italy)
Locally quasi-convex compatible topologies on a topological group (page 47)

18:00 – 18:30 Jorge Picado (University of Coimbra, Portugal)
Strict insertion of continuous real functions in pointfree topology (page 40)

18:30 – 18:50 Valentín Gregori (Universidad Politécnica de Valencia, Spain)
Completion of stationary fuzzy metric spaces (page 55)

21:30 *Conference dinner*

Friday September 10, 2010

Morning session

09:30 – 10:00 Miroslav Husek (Charles University, University J.E. Purkyne, Czech Republic)
Extensions of mappings and pseudometrics (page 39)

10:00 – 10:20 Joan Gerard Camarena (Universidad Politécnica de Valencia, Spain)
Fuzzy metrics for image processing (page 46)

10:20 – 10:40 Almanzor Sapena (Universidad Politécnica de Valencia, Spain)
Metrics and fuzzy metrics (page 70)

10:40 – 11:00 Hans-Peter Kunzi (University of Cape Town, South Africa)
Permutable quasi-uniformities (page 58)

11:00 - 11:30 *Coffee break*

11:30 – 11:50 Mohamed Bakier (Assiut University, Egypt)
Fuzzy multivalued functions between fuzzy minimal spaces (page 45)

11:50 – 12:40 Angelo Bella (University of Catania, Italy)
On selective separability (page 21)

12:40 – 13:00 Iraide Mardones (Universidad del País Vasco-Euskal Herriko Unibertsitatea, Spain)
On insertion and extension of lattice-valued functions on preordered topological spaces (page 61)

Friday September 10, 2010

Afternoon session

16:00 – 16:30	Salvador Hernández (Universitat Jaume I, Spain) Bounded sets in topological groups (page 38)
16:30 – 16:50	Tero Vedenjuoksu (University of Oulu, Finland) The stone-cech compactification of topological groups (page 73)
16:50 – 17:10	Antonio A. Pulgarín (Universidad de Extremadura, Spain) General equilibrium in $C(P)$-type economies (page 65)
17:10 – 17:40	*Coffee break*
17:40 – 18:30	Jesús A. Jaramillo (Universidad Complutense de Madrid, Spain) Approximation of lipschitz functions and algebras of diferentiable functions on finsler manifolds (page 26)
18:30	*Closing Ceremony*

Main Talks

Badajoz (Spain)

September 7 – 10, 2010

Universidad de Extremadura, Badajoz
FRIDAY 10, 11:50 – 12:40

On selective separability

Angelo Bella

University of Catania, Italy
e-mail: bella@dmi.unict.it

Abstract

A space X is selectively separable if for any countable family $\{D_n : n < \omega\}$ of dense subsets of X we may select finite sets $F_n \subseteq D_n$ in such a way that the set $\bigcup\{F_n : n < \omega\}$ is dense in X.

M. Scheepers was the first to consider this notion, during his investigation on selection principles and topological games. Then, a systematic study was initiated in the seminal paper [BBMT]. After this paper, several authors started to consider selectively separable spaces and other related notions. For instance, G. Gruenhage and M. Sakai solved some problems posed in [BBMT] and A. Dow presented several interesting and deep results during the last two Summer Topological Conferences (Brno, 2009 and Kielce, 2010).

In this talk we will describe the basic facts on selective separability. Then we will focus on the following elegant and quite unexpected Dow-Barman's result:

Proposition 1. *A T_2 separable Frechet space is selectively separable.*

There are two natural ways to strengthen the above result: one is by moving from Frechet to radial and the other is by moving from Frechet to sequential. Recall that a space X is radial if for any set $A \subseteq X$ and any $x \in \overline{A}$ there exists a well-ordered net $\{x_\alpha : \alpha < \kappa\} \subseteq A$ which converges to x.

We will show that the strengthening is possible in the first case, but in general not in the second one. In the remaining part of the talk, we will discuss the possibility to generalize the following:

Proposition 2. *A T_3 separable countably compact space of countable tightness is selectively separable.*

References

[BBMT] A. Bella, M. Bonanzinga, M. Matveev, V. Tkachuk, Selective separability: general facts and behaviour in countable spaces, *Topology Proceedings* **32** (2008), 15 – 30.

Universidad de Extremadura, Badajoz
TUESDAY 7, 10:10 – 11:00

Action, uniformity and proximity

A. Di Concilio

University of Salerno, Italy
e-mail: diconci@unisa.it

AMS *Subject Classification* (2010): 54C35, 57S05, 54H99

Abstract

The "*incipit*" of the homeomorphism group theory resides in the early seminal paper by G. Birkoff [2]. In that paper with jointly an apparent simplicity and an impressive bright proof strategy, Birkoff positively answered to the query: *There exists a topology on the full self-homeomorphism group of a compact metric space which makes it into a topological group and a subspace of the Hilbert cube?* His investigation involved three convergence options: pointwise convergence, continuous convergence, g-convergence (continuous convergence in both directions). For the Euclidean real numbers space and all real intervals, and then for all connected one-dimensional manifolds, he proved the adequacy of pointwise topology in providing continuity of both the group operations, product and inverse function. Furthermore, he proved the inadequacy of the pointwise topology for any Euclidean space, hence for any locally Euclidean space, of dimension greater than one and the adequacy of the continuous convergence, which in that case is also topological, for all Euclidean spaces and their locally connected open subspaces and, more generally, for all O-spaces.

Later on, in [1] R. Arens focused on those topologies on the full self-homeomorphism group $H(X)$ of a Tychonoff space X which yield continuity of both the group operations and, at the same time, yield continuity of the evaluation function $e : (f, x) \in H(X) \times X \to f(x) \in X$ and posed the problem of the existence of the least element in the upper-semilattice (ordered by the usual inclusion) $L_H(X)$ of all topologies with these two features, that he called *admissible group topologies*. For T_2 locally compact spaces Arens

proved the existence of the least element in $L_H(X)$, the *g-topology*, which, under the additional property of local connectedness for X, just agrees with the compact-open topology.

Of course, there are many different ways to topologize the full self-homeomorphism group. For instance, it can be endowed with the subspace topology induced by any of all known function space topologies. Nevertheless, following Birkoff and Arens, we also focus on admissible group topologies that, as a matter of fact, are *those ones which determine a group action of $H(X)$ on X*.

The existence of a least element in $L_H(X)$ has been proven for T_2 rim-compact and locally connected spaces in [3] and in [4] for products of T_2 zero-dimensional spaces each satisfying the property: *any two non-empty clopen subspaces are homeomorphic*, and, as a corollary, for every zero-dimensional metrizable space of diversity one such as, for instance, the rationals, the irrationals, ther Baire spaces. As rim-compactness is a weak and peripherical compactness property one might think any further relaxation as impossible, but X being rim-compact is not a necessary condition in order for a least admissible group topology to exist. In fact, as shown in [5], the space $R \times Q$ with the product topology, where R and Q are the sets of the real and rational numbers respectively, both carrying the Euclidean topology, is not rim-compact, but anyway $L_H(R \times Q)$ admits a least element. At a first time, significant examples strongly suggest to search the admissible group topologies by means of a compact extension procedure by investigating those uniform topologies on $H(X)$ deriving from a totally bounded uniformity on X whose uniform completion is a T_2-compactification of X to which any self-homeomorphism continuously extends. But then also a new method, other than the compact extension procedure, the construction of the fine group topology associated with a given class of compatible metrics on X works well.

For T_2 locally compact spaces X, the compact-open topology on $H(X)$, which is also the topology of uniform convergence on compacta derived from any uniformity on X, is admissible and yields continuity of the product function. Unfortunately in general, the compact-open topology does not provide continuity of the inverse function. But, with the following additional property: *any point of X has a compact connected neighbourhood*, due to J.J. Dijkstra in [8], the compact-open topology becomes a group topology and, as a consequence, the least admissible group topology of $H(X)$. Accordingly, the compact-open topology on $H(X)$ quotes as the most eligible one if X is a manifold of finite dimension or X is an infinite dimensional manifold modelled on the Hilbert cube. In looking for topologies of uniform convergence on members of a given family, containing all compact sets, which are admissible group topologies, one can focus beyond local compactness. In order to do so, it appears as suggestive example that of bounded sets of an infinite dimensional normed vector space carrying as proximity the metric proximity associated with the norm. Because of the local compactness of X is equivalent to the family of compact sets of X being a *boundedness* of X, which, jointly any Efremovic proximity of X, gives a *local proximity space*, this particular case falls within the more general one in which compact sets are substituted with bounded sets in a local proximity space, while the previous Dijkstra-property is replaced by the following one: *for each non-empty bounded set B there exists a finite number of connected bounded sets $B_1, \dots, B_n$ such that B is strongly contained* $\mathrm{int}(B1) \cup \cdots \cup \mathrm{int}(Bn)$. So doing, analogous results have been achieved in [6], also recasting topologies of uniform convergence on members of a network as proximal set-open topologies.

In [9], again for T_2 locally compact spaces, K.R. Wicks gives a necessary and sufficient condition for the compact-open topology to be a group topology, by using methods of non-standard analysis and action on hyperspaces. The generalization of the Wicks's result to hit and miss topologies on hyperspaces and proximal set-open topologies on homeomorphisms is contained in the manuscript, as a work in progress, [7].

References

[1] R. Arens, Topologies for homeomorphism groups, *Amer. J. Math.* **68** (1946), 593 – 610.

[2] G. Birkoff, The topology of transformation sets, *Annals of Math.* **35** (4) (1934), 861 – 875.

[3] A. Di Concilio, Topologizing homeomorphism groups of rim-compact spaces, *Topology Appl.* **153** (2006), 1867 – 1885.

[4] A. Di Concilio, Group action on 0-dimensional spaces, *Topology Appl.* **154** (2007), 2050 – 2055.

[5] A. Di Concilio, Action on $R \times Q$ and fine group topologies, *Topology Appl.* **156** (2009), 956 – 962.

[6] A. Di Concilio, Action, uniformity and proximity, *Quaderno di Matematica* **22** SUN (2009), 71 – 88.

[7] A. Di Concilio, Action on hyperspaces, *manuscript.*

[8] J.J. Dijkstra, On homeomorphism groups and the compact-open topology, *Amer. Math. Monthly* **112** (2005), 910 – 912.

[9] K.R. Wicks, Topologicality of groups of homeomorphisms, *unpublished.*

Universidad de Extremadura, Badajoz
FRIDAY 10, 17:40 – 16:30

Approximation of Lipschitz functions and algebras of differentiable functions on Finsler manifolds

Jesús A. Jaramillo

Departamento de Análisis Matemático, Universidad Complutense de Madrid
28040 *Madrid, Spain*
e-mail: jaramil@mat.ucm.es

AMS *Subject Classification* (2010): 58C05, 54C35, 46E25

Abstract

We report here on a joint work with M.I. Garrido and Y. Rangel. We first consider the problem of smooth approximation of Lipschitz functions on a Finsler manifold M, and we obtain the following result. Let $f : M \to \mathbb{R}$ be a Lipschitz function, and denote by $\mathrm{Lip}(f)$ its Lipschitz constant. Then for each $\varepsilon > 0$, there exists a C^1 and Lipschitz function $g : M \to \mathbb{R}$ such that $|f(x) - g(x)| \leq \varepsilon$ for all x in M and $\mathrm{Lip}(g) \leq \mathrm{Lip}(f) + \varepsilon$. Some applications of this result are also given. In particular, as a consequence we obtain an analog of the Myers-Nakai theorem in the context of absolutely homogeneous Finsler manifolds. Namely, we prove that, in this case, the metric structure of the manifold M is determined by the Banach algebra structure of the space $C_b^1(M)$ of bounded C^1 functions on M which have bounded derivative, endowed with its natural norm.

Universidad de Extremadura, Badajoz
THURSDAY 9, 12:10 – 13:00

An overview of complexity spaces and its applications to Computer Science

Oscar Valero

Departamento de Ciencias Matemáticas e Informática
Universidad de las Islas Baleares
Carretera de Valldemossa km. 7.5, 07122 *Palma de Mallorca, Spain*
e-mail: o.valero@uib.es

AMS *Subject Classification* (2010): 54E50, 5H25, 47H50, 54H99, 68Q25, 68Q45, 68Q55

Abstract

In 1995 M.P. Schellekens introduced the theory of (quasi-metric) complexity spaces in order to obtain a common mathematical foundation for Denotational Semantics and Complexity Analysis in Computer Science [Electr. Notes in Theor. Comput. Sci. 1 (1995), 211 – 232]. This theory is based on the notion of a complexity distance, actually a quasi-metric which reveals when a program is more efficient than another one. The relevance of complexity spaces theory is given by the fact that it allows to apply fixed point techniques of Denotational Semantics to Complexity Analysis of algorithms and programs. In fact, the applicability of this new theory to the asymptotic complexity analysis of Divide and Conquer algorithms was illustrated by Schellekens. In particular, he gave a new proof, based on the use of the celebrated Banach fixed point theorem, of the well-known fact that Mergesort algorithm has optimal asymptotic average running time of computing.

Later on, S. Romaguera and Schellekens introduced and studied a new complexity structure which was called the dual complexity space and remains valid for the asymptotic complexity analysis of algorithms [Topology Appl. 98 (1999), 311 – 322]. This new structure has more mathematical robust properties than the original one. In particular, and contrarily to the case of the complexity space, the dual complexity space is a pointed ordered cone en-

dowed, also, with a quasi-metric as complexity distance. Nevertheless, the dual space (as in the case of the original complexity space) is not suitable to give quantitative measure of the improvement in complexity obtained when a program is replaced by another one. With the aim of avoiding this handicap Romaguera, E.A. Sánchez-Pérez and O. Valero introduced a new dual complexity space [Kybernetika 39 (2003), 569 − 582], which is also a pointed ordered cone but the complexity distance is an extended quasi-metric which has nice topological properties as, for instance, Hausdorffness.

More recently, motivated by the connection between Denotational Semantics and the original complexity space, Romaguera and Valero have gone more deeply into the aforementioned relationship and they have constructed a new complexity space by using partial functions [Int. J. Comput. Math. 85 (2008), 631 − 640]. Such a space has pointed ordered cone structure and it can be also endowed with an extended quasi-metric as a complexity distance. Furthermore, Romaguera and Valero have shown that the complexity space of partial functions is an appropriate framework to model at the same time processes that arise in natural way in symbolic computation, complexity analysis and denotational semantics. As an illustrative example they have shown, among other things, that it is suitable to check the correctness of denotational specifications of recursive programs.

Since the interest in dual complexity spaces has increased in the last years and their study constitutes a part of the interdisciplinary research on Computer Science and Topology, we will give a general survey of this kind of spaces and their applications to Computer Science.

Invited Talks

Badajoz (Spain)

September 7 – 10, 2010

Universidad de Extremadura, Badajoz
TUESDAY 7, 18:00 – 18:30

On the periodic-recurrent property on dendrites and dendroids

Francisco Balibrea

Universidad de Murcia, Spain
e-mail: balibrea@um.es

AMS *Subject Classification* (2010): 37B45, 37B20

Abstract

We say a continuum $\mathbb{X}$ has the Periodic-Recurrent property (PR-property) if $\overline{P(f)} = \overline{R(f)}$ for any continuous map from $\mathbb{X}$ into itself, where $P(f)$ and $R(f)$ denote respectively the set of periodic and recurrent points of the dynamical system $(\mathbb{X}, f)$.

The PR-property indicates that all interesting dynamical behaviors of the system occurs on $\overline{P(f)}$, for example, every minimal set is contained in it, $h(f) = h(f|\overline{P(f)})$ where h denotes the topological entropy and $\mu(\overline{P(f)}) = 1$ for every normalized invariant measure and no smaller closed invariant subset has this property. Additionally, the PR-property is connected to other characteristics: *depth of the center, expansiveness* and the set of ω-limit points of the system.

In this talk we will survey some known results on *dendrites, graphs, arc-like and tree-like continua* and give some others new and concentrate in the characterization of those dendrites having the PR-property (those not containing a topological copy of the Gehman dendrite). Additionally we will studied the relationship between the PR-property and the chaoticity in Devaney's sense.

Universidad de Extremadura, Badajoz
THURSDAY 9, 9:30 – 10:00

Topologies corresponding to continuous representability of preorders

Gianni Bosi

Università di Trieste, Dipartimento di Scienze Economiche Aziendali, Matematiche e Statistiche "Bruno de Finetti" Piazzale Europa 1, 34127 *Trieste, Italy*
e-mail: giannibo@econ.units.it

Key words and phrases: weakly continuous preorder, weakly continuous representation property, continuous representation property.

AMS *Subject Classification* (2000): 54F05, 91B16, 06A05

Abstract

In this paper I present conditions on a topology τ on a fixed nonempty set X under which every *weakly continuous* (*weakly upper semicontinuous*) preorder $\precsim$ on the topological space (X, τ) is representable by a continuous (upper semicontinuous) *order preserving* function (*utility function*) $u : (X, \precsim, \tau) \to (\mathbb{R}, \leq, \tau_{nat})$.

Weakly continuous preorders were first introduced by Herden and Pallack [8] and then studied by other authors (see e.g. Bosi and Herden [2, 3] and Bosi, Caterino and Ceppitelli [1]) in order to generalize the classical concept of *continuity* of a total preorder to the case of a not necessarily total preorder on a topological space.

A topology τ on X is said to satisfy the *Weakly Continuous Representation Property* (the *Continuous Representation Property*) if every weakly continuous (every continuous total) preorder $\precsim$ on (X, τ) admits a continuous order preserving function $u : (X, \precsim, \tau) \to (\mathbb{R}, \leq, \tau_{nat})$.

The Continuous Representation Property was introduced by Herden [6] and then studied by Herden and Pallack [7], Campión, Candeal and Induráin [4] and Campión, Candeal, Induráin and Mehta [5]. It is known that a topology τ on a set X satisfies the weakly continuous representation property whenever

the product topology $\tau \times \tau$ on $X \times X$ is *hereditarily Lindelöf* (see Bosi, Caterino and Ceppitelli [1, Theorem 3.1]). This is the case when τ has a *countable network weight*.

The *Weakly Semicontinuous Representation Property* is also considered in the paper.

References

[1] G. Bosi, A. Caterino, R. Ceppitelli, Existence of continuous utility functions for arbitrary binary relations: some sufficient conditions, *Munich Personal RePEc Archive*, 2009 (submitted).

[2] G. Bosi, G. Herden, On a possible continuous analogue of the Szpilrajn theorem and its strengthening by Dushnik and Miller, *Order* **23** (2006), 271 – 296.

[3] G. Bosi, G. Herden, Utility representations and linear refinements of arbitrary binary relations, preprint, 2007.

[4] M.J. Campión, J.C. Candeal, E. Induráin, Preorderable topologies and order-representability of topological spaces, *Topology and its Applications* **156** (2009), 2971 – 2978.

[5] M.J. Campión, J.C. Candeal, E. Induráin, G.B. Mehta Representable topologies and locally connected spaces, *Topology and its Applications* **154** (2007), 2040 – 2049.

[6] G. Herden, Topological spaces for which every continuous total preorder can be represented by a continuous utility function, *Mathematical Social Sciences* **22** (1991), 123 – 136.

[7] G. Herden, A. Pallack, Useful topologies and separable systems, *Applied General Topology* **1** (2000), 61 – 82.

[8] G. Herden, A. Pallack, On the continuous analogue of the Szpilrajn Theorem I, *Mathematical Social Sciences* **43** (2002), 115 – 134.

Universidad de Extremadura, Badajoz
THURSDAY 9, 16:00 – 16:30

Nonlinear Banach-Stone theorems for lattices of Lipschitz functions

Félix Cabello Sánchez [1]

Departamento de Matemáticas, Universidad de Extremadura
06071 *Badajoz, Spain*
e-mail: fcabello@unex.es

Co-author: Javier Cabello Sánchez

AMS *Subject Classification* (2010): 46Txx; 46E05

Abstract

By a Banach-Stone theorem we mean the statement that certain (often algebraical) structure of a system of (continuous, real-valued) functions determines some additional (often topological) structure on the underlying topological space. As everyone knows the genuine Banach-Stone theorem says that two compact spaces are homeomorphic provided their corresponding spaces of continuous functions are isometric in the natural supremum norm.

We survey a number of classical results. These include venerable oldies by Kaplansky and Shirota and further developments by Namioka-Saeki and Lochan-Strauss, which provide the necessary nonlinear background. Then we present some recent results for lattices of uniformly continuous and Lipschitz functions without any linearity assumption. Sample result: two complete metric spaces of finite diameter are Lipschitz homeomorphic if (and only if, of course) the corresponding lattices of Lipschitz functions are isomorphic. Here, a lattice isomorphism is just a bijection preserving the order in both directions, in particular linearity is not assumed.

[1] http://kolmogorov.unex.es/~fcabello

References

[1] G. Birkhoff, "Lattice Theory", corrected reprint of the 1967 third edition, American Mathematical Society Colloquium Publications 25, American Mathematical Society, Providence, R.I., 1979.

[2] F. Cabello Sánchez, J. Cabello Sánchez, Nonlinear isomorphisms of lattices of Lipschitz functions, *Houston J. Math* (to appear in 2010).

[3] S. Cater, Remark on a result of Kaplansky concerning $C(X)$, *Michigan Math. J.* **12** (1965), 97 – 103.

[4] M.I. Garrido, J.A. Jaramillo, Variations on the Banach-Stone theorem, *Extracta. Math.* **17** (3) (2002), 351 – 385.

[5] M. Henriksen, On the equivalence of the ring, lattice, and semigroup of continuous functions, *Proc. Amer. Math. Soc.* **7** (1956), 959 – 960.

[6] T. Itô, On the continuity of lattice automorphisms on continuous function lattices, *Illinois J. Math.* **8** (1964), 419 – 424.

[7] I. Kaplansky, Lattices of continuous functions, *Bull. Amer. Math. Soc.* **53** (1947), 617 – 623.

[8] I. Kaplansky, Lattices of continuous functions II, *Amer. J. Math.* **70** (1948), 626 – 634.

[9] R. Lochan, D. Strauss, Lattice homomorphisms of spaces of continuous functions, *J. London Math. Soc. (2)* **25** (2) (1982), 379 – 384.

[10] I. Namioka, S. Saeki, On lattice isomorphisms of $C(X)^+$, *Tokyo J. Math.* **1** (2) (1978), 345 – 368.

[11] T. Shirota, A generalization of a theorem of I. Kaplansky, *Osaka Math. J.* **4** (1952), 121 – 132.

Universidad de Extremadura, Badajoz
TUESDAY 7, 16:00 – 16:30

Domain-valued functions vs scales

María Angeles de Prada Vicente

Departamento de Matemáticas, Universidad del País Vasco (UPV-EHU)
Apto. 644, 48080 *Bilbao, Spain*
e-mail: mariangeles.deprada@ehu.es

Co-authors: Javier Gutiérrez García, Tomasz Kubiak

AMS *Subject Classification* (2010): 54D15, 06B35, 54G05

Abstract

The purpose of this talk is to introduce a method which allows to substitute, succesfully, functions in a topological space with values in a bounded complete domain by “certain” families of subsets of the space. These families of subsets will be called scales (prescales).

Some order-like and topological aspects (among them lower and upper limits) of such functions are characterized in terms of scales. This devel¡opment is then used to give a sufficient condition for inserting a Lawson continuous function between two comparable domain-valued functions.

As an application, new characterizations of some well-known classes of topological spaces will be given.

Universidad de Extremadura, Badajoz
TUESDAY 7, 12:30 – 13:00

The algebra of those continuous functions on X that are locally in a subalgebra of $C(X)$

Jesús M. Domínguez

Universidad de Valladolid, Spain
e-mail: jmd@agt.uva.es

Co-authors: C. Lorenzo, M.A. Mulero

AMS *Subject Classification* (2010): 54C40

Abstract

Let X be a Tychonoff space and let $C(X)$ be the algebra of all real-valued continuous functions on X. For A a subalgebra of $C(X)$, we shall denote by $\mathrm{L}\,A$ the subalgebra of $C(X)$ consisting of those functions on X that are locally in A, and we shall say that the algebra A is local on X if $\mathrm{L}\,A = A$. When one tries to characterize $C(X)$ among its subalgebras, it is natural to consider local algebras on X that contain the constant functions and separate points and closed subsets of X. It is also natural to assume that the algebras are closed under uniform convergence and inversion. In this talk we shall show some relationships between these properties.

Universidad de Extremadura, Badajoz
FRIDAY 10, 16:00 – 16:30

Bounded sets in topological groups

Salvador Hernández

Departamento de Matemáticas, Universitat Jaume I
12071 *Castellón, Spain*
e-mail: hernande@mat.uji.es

Co-authors: C. Chis, M.V. Ferrer, B. Tsaban

AMS *Subject Classification* (2010): 22A05, 22D35, 43A40, 54A25, 54H11

Abstract

We report on some recent results about the structure of different classes of bounded sets on a metrizable topological group. Among other findings, the cofinality of different systems of bounded sets is calculated and some applications are given to the topological structure of the group. Along this line, we apply methods from Pontryagin duality in order to estimate the character (minimal cardinal of a neighbourhood base of the identity) for several classes of topological abelian groups.

Universidad de Extremadura, Badajoz
FRIDAY 10, 9:30 – 10:00

Extensions of mappings and pseudometrics

Miroslav Hušek

Charles University, University J.E. Purkyne, Czech Republic
e-mail: mhusek@karlin.mff.cuni.cz

AMS *Subject Classification* (2010): 54C20

Abstract

This talk will present a survey on extensions of maps and pseudometrics, their relations and equivalences.

Probably the first occurence in literature of a continuous extension of a continuous mapping from a subspace to the whole space was the paper of H. Lebesgue from 1907. We shall trace the various methods (now mostly forgotten) of followers of Lebesgue (like Tietze, Hausdorff, Brouwer, Poussin, Bohr, Urysohn, Isbell, Katětov and others) who sometimes implicitly proved more than they formulated. Sometimes a hidden result was reproved or generalized several decades later without knowing older methods.

For instance, uniformly continuous extensions (for metric spaces) could be known around 1920, as well as extensions of continuous maps into topological linear spaces. Some unpublished results and methods by F. Hausdorff will be mentioned.

Universidad de Extremadura, Badajoz
THURSDAY 9, 18:00 – 18:30

Strict insertion of continuous real functions in pointfree topology

Jorge Picado

Department of Mathematics, University of Coimbra, Portugal
e-mail: picado@mat.uc.pt

Co-author: Javier Gutiérrez García [2]

AMS *Subject Classification* (2010): 06D22, 13J25, 54C30, 54D15

Abstract

As is well known, many results in classical topology are actually consequences of corresponding facts in pointfree topology, that is, in the setting of frames and locales, the lattices (introduced in the late 1950s in the Seminaire Ehresmann) which may be viewed as abstractly defined lattices of open sets of spaces. In collaboration with J. Gutiérrez García, T. Kubiak, M.J. Ferreira and B. Banaschewski we have recently obtained several results of this kind in the realm of real-valued functions ([1, 3, 4, 5, 6, 7, 8, 9, 11]). In this talk we report on the case of strict insertion of functions, obtained as an application of our results on rings of pointfree real functions ([10]). This extends the classical insertion theorem of Dowker ([2]) regarding normal countably paracompact spaces.

References

[1] B. Banaschewski, J. Gutiérrez García, J. Picado, Extended real functions in pointfree topology, in preparation.

[2] C.H. Dowker, On countably paracompact spaces, *Canad. J. Math.* **3** (1951), 219 – 224.

[3] M.J. Ferreira, J. Gutiérrez García, J. Picado, Completely normal frames and real-valued functions, *Topology Appl.* **156** (2009), 2932 – 2941.

[2] UPV-EHU, Bilbao, Spain.

[4] M.J. Ferreira, J. Gutiérrez García, J. Picado, Insertion of continuous real functions on spaces, bispaces, ordered spaces and pointfree spaces — a common root, *Appl. Categ. Structures*, in press.

[5] J. Gutiérrez García, T. Kubiak, General insertion and extension theorems for localic real functions, *J. Pure Appl. Algebra*, in press.

[6] J. Gutiérrez García, T. Kubiak, J. Picado, Monotone insertion and monotone extension of frame homomorphisms, *J. Pure Appl. Algebra* **212** (2008), 955 – 968.

[7] J. Gutiérrez García, T. Kubiak, J. Picado, Lower and upper regularizations of frame semicontinuous real functions, *Algebra Universalis* **60** (2009), 169 – 184.

[8] J. Gutiérrez García, T. Kubiak, J. Picado, Pointfree forms of Dowker's and Michael's insertion theorems, *J. Pure Appl. Algebra* **213** (2009), 98 – 108.

[9] J. Gutiérrez García, T. Kubiak, J. Picado, Localic real-valued functions: a general setting, *J. Pure Appl. Algebra* **213** (2009), 1064 – 1074.

[10] J. Gutiérrez García, J. Picado, Rings of real functions in Pointfree Topology, *Preprint DMUC 10-08*, February 2010 (submitted for publication).

[11] J. Picado, A new look at localic interpolation theorems, *Topology Appl.* **153** (2006), 3203 – 3218.

Contributed Talks

Badajoz (Spain)

September 7 – 10, 2010

Universidad de Extremadura, Badajoz
FRIDAY 10, 11:30 – 11:50

Fuzzy multivalued functions between fuzzy minimal spaces

M.Y. Bakier

Mathematics Department, Assiut University, Assiut, Egypt
e-mail: mybakier@yahoo.com

Key words and phrases: minimal spaces, fuzzy multifunction, fuzzy m-compact, fuzzy m-connected.

AMS *Subject Classification* (2010): 54A40

Abstract

The biggest difference between fuzzy functions and fuzzy multivalued functions (multifunction) has to do with the definition of an inverse image. For a fuzzy multivalued function there are two type of inverses. These two definitions of the inverse then leads to two definitions of continuity. In this paper, the fuzzy upper and lower continuous multifunction between a fuzzy minimal spaces have been presented. Certain characterizations and several properties of these fuzzy multifunction are obtained. Moreover, we define fuzzy minimal compactness and fuzzy minimal connected, we investigate some of their properties.

Universidad de Extremadura, Badajoz
FRIDAY 10, 10:00 – 10:20

Fuzzy metrics for image processing

Joan Gerard Camarena Estruch

Instituto Universitario de Matemática Pura y Aplicada
Universidad Politécnica de Valencia
Camino de Vera s/n, 46022 *Valencia, Spain*
e-mail: jcamarena@dsic.upv.es

Co-author: Samuel Morillas

AMS *Subject Classification* (2010): 68U10, 54A40

Abstract

Novel mathematical concepts are interesting for Engineering problems if they provide new or improved solutions. In this talk, we will show how fuzzy metrics are useful to design better solutions to a problem in the image processing field: image filtering. First, we will review the basic concepts related to fuzzy metrics and we will provide a series of examples of fuzzy metrics, including a novel fuzzy metric specifically design to detect impulse noise in colour images. Later, we use these fuzzy metrics for colour image filtering by means of a vector ordering approach. We show that the obtained results are promising.

Universidad de Extremadura, Badajoz
THURSDAY 9, 17:40 – 18:00

Locally quasi-convex compatible topologies on a topological group

Dikran Dikranjan

Department of Mathematics and Computer Science
University of Udine, 33100 *Udine, Italy*
e-mail: dikranja@dimi.uniud.it

Co-authors: Lydia Aussenhofer, Elena Martín-Peinador

AMS *Subject Classification* (2010): 54C40, 14E20, 46E25, 20C20

Abstract

For a locally quasi-convex abelian group (G,τ) we study the poset $\mathcal{LQC}(G)$ of all locally quasi-convex topologies on G that are compatible (i.e., have the same Pontryagin dual as (G,τ)) ordered by inclusion. Sometimes $\mathcal{LQC}(G)$ may collaps to a singleton (e.g., when G is pseudocompact), but in general the structure of this poset seems to be highly complicated. While it obviously has a bottom element, namely the weak topology $\sigma(G,\widehat{G})$, already the question of whether it has also a top element (called Mackey topology) is still open.

This talk will discuss the case when the group G is locally compact. When G is not σ-compact, we obtain a satisfactory description of the poset $\mathcal{LQC}(G)$. Namely, we show that it is quasi-isomorphic to the poset $\mathbf{Fil}_X$ of all free filters on an approriate set X. In this way the computation of most of the cardinal invariants of $\mathcal{LQC}(G)$ (as size, height, depth, width, maximum size of chains) can be carried out in the poset $\mathbf{Fil}_X$ of combinatorial nature, i.e., reduced to purely set-theoretical questions.

Universidad de Extremadura, Badajoz
THURSDAY 9, 10:20 – 10:40

On pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces

Erdal Ekici

Department of Mathematics
Canakkale Onsekiz Mart University Terzioglu Campus
17020 *Canakkale, Turkey*
e-mail: eekici@comu.edu.tr

Key words and phrases: pre-I-open set, semi-I-open set, b-I-open set, ideal topological space, decomposition, closure operator.

AMS *Subject Classification* (2010): 54A05, 54A10, 54C08, 54C10

Abstract

The aim of this paper is to investigate some properties of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces. Some relationships of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces are discussed. Moreover, decompositions of continuity are provided.

Universidad de Extremadura, Badajoz
THURSDAY 9, 11:50 – 12:10

Applications of fractal dimensions for fractal structures

MANUEL FERNÁNDEZ-MARTÍNEZ

Área de Geometría y Topología, Universidad de Almería (UAL)
04071 *Almería, Spain*
e-mail: fmm124@ual.es

Co-author: M.A. SÁNCHEZ-GRANERO

AMS *Subject Classification* (2010): 28A80, 54E35, 68Q55

ABSTRACT

The study and analysis of fractals has become more and more important during last years since the increasing number of applications to diverse fields that these kind of sets have experimented. In this way, the introduction of *fractal structures* has allowed to formalize some topics on fractal theory from both theoretical and applied points of view.

In particular, one of the main tools applied in order to study fractals is the fractal dimension, understood as the classical *box-counting* and Hausdorff dimensions, since it is a single quantity which offers some useful information about the complexity of the set under study. These two notions can be defined for any metrizable space, and while the former is *better* from an applied point of view, the latter presents *better* analytical properties, though it can result very difficult or impossible to calculate in practical applications.

The main purpose of this talk consists of showing some interesting applications of those definitions of fractal dimension that we introduced in previous works in order to calculate this quantity for any subset with respect to any fractal structure. Thus, on the one hand, recall that fractal dimensions I & II generalize the box-counting dimension on the more general context of GF-spaces. Its easiness of effective calculation allows to use them in some contexts where the box-counting scheme cannot be applied, like the domain of words. In this way, we show how to compute and understand the fractal dimension of

a language generated by means of a regular expression, where infinite length words could exist. Moreover, we apply these definitions in order to calculate the fractal dimension of any natural language which leads to quantify its complexity and analyze the variety of words used in any text written in that language. It also allows to compare the complexity of any translation of a text respect to its original version.

On the other hand, the fractal dimension III constitutes a suitable discretization of the theoretical model underlying in Hausdorff measure and dimension, which can be effectively calculated on an easy way. Taking it into account, we show an interesting application of fractal dimension III to the study of the complexity of a curve by means of an induced fractal structure on its image set, which leads to more accurate information about it than classical fractal dimension definitions provide, since our method takes into account its structure as well as the complexity of the way it was constructed.

Universidad de Extremadura, Badajoz
THURSDAY 9, 10:40 – 11:00

Insertion of real functions in completely normal point-free spaces

Maria João Ferreira

CMUC, Departamento de Matemática, Universidade de Coimbra
3001-454 *Coimbra, Portugal*
e-mail: mjrf@mat.uc.pt

Co-authors: Javier Gutiérrez García, Jorge Picado

AMS *Subject Classification* (2010): 06D22, 26A15, 54C30, 54D15

Abstract

The setting introduced in [2], by Gutierrez Garcia, Kubiak and Picado, for dealing with arbitrary, not necessarily continuous, point-free real functions gives point-free topology the freedom to deal with general real functions only available before to point-set topology. As an illustration of this we show in this talk how completely normal frames can be characterized in terms of an insertion result for general real functions ([1]). This characterization extends the following well-known classical result of T. Kubiak ([3]) for general functions $f_1, f_2 : X \to \mathbb{R}$:

If ${f_1}^- \leq f_2$ *and* $f_1 \leq {f_2}^\circ$*, then there exists a lower semicontinuous* $f : X \to \mathbb{R}$ *such that* $f_1 \leq f \leq f^- \leq f_2$ *(where* ${f_1}^-$ *denotes the upper regularization of* f_1 *and* ${f_2}^\circ$ *denotes the lower regularization of* f_2*).*

References

[1] M.J. Ferreira, J. Gutiérrez García, J. Picado, Completely normal frames and real-valued functions, *Topology Appl.* **156** (2009), 2932 – 2941.

[2] J. Gutiérrez García, T. Kubiak, J. Picado, Localic real-valued functions: a general setting, *J. Pure Appl. Algebra* **213** (2009), 1064 – 1074.

[3] T. Kubiak, A strengthening of the Katětov-Tong insertion theorem, *Comment. Math. Univ. Carolinae* **34** (1993), 357 – 362.

Universidad de Extremadura, Badajoz
TUESDAY 7, 17:40 – 18:00

The conditioning of the winding number of plane curves

Juan-Luis García Zapata

Departamento de Matemáticas, Universidad de Extremadura
Avenida de Elvas, 06071 *Badajoz, Spain*
e-mail: jgzapata@unex.es

AMS *Subject Classification* (2010): 65J20, 54H99, 54C35

Abstract

The condition number of a continuous map $f : D \to Y$ between metric spaces in a point $x_0 \in X$ is $\operatorname{cond}(f, x_0) = \limsup_{x \to x_0} \frac{d_X(x, x_0)}{d_Y(f(x), f(x_0))}$. Is a notion useful in Numerical Analysis, where X is a space of problems (for example, the set of $n \times n$ linear systems of equations), Y the space of solutions, D the problems with solution and $S = D^c$ the set of singular problems. Apart from its application to error analysis, the condition number can give light about f. For example, in certain settings, it is verified that $\operatorname{cond}(f, x_0) = \frac{1}{d_X(x_0, S)}$ (Eckart-Young theorem [1]).

In this talk we display a similar result when X is the set of plane curves and f the winding number (the number of times a curve surround the origin). As $Y = \mathbb{Z}$ the classical definition is not relevant. We use the statement of Eckart-Young as definition of the condition number, and show its use in error analysis. This emphasize the current view of the conditioning as a metric property, not depending on the parameters of the problem [2].

References

[1] S. Smale, Complexity theory and numerical analysis, *Acta Numerica* **6** (1997), 523 – 551.

[2] F. Cucker, T. Krick, G. Malajovich, M. Wschebor, A numerical algorithm for zero counting. II: Distance to ill-posedness and smoothed analysis, *Journal of Fixed Point Theory and Applications* **6** (2009), 285 – 294.

Universidad de Extremadura, Badajoz
TUESDAY 7, 16:50 – 17:10

On a new relative invariant covering dimension [3]

D.N. Georgiou

University of Patras, Department of Mathematics, 265 04 *Patras, Greece*
e-mail: georgiou@math.upatras.gr

Co-author: A.C. Megaritis

AMS *Subject Classification* (2010): 54B99, 54C25

Abstract

In [1] (see also [2] page 35) two relative covering dimensions, denoted by dim and dim*, defined and studied. Here we give and study a new relative covering dimension, denoted by r-dim, which is different from dim and dim*. Finally, we give some questions concerning the new relative dimension r-dim.

References

[1] J. Valuyeva, On relative dimension concepts, Q & A in General Topology, Vol. 15, 1997.

[2] K.P. Hart, Jun-iti Nagata, J.E. Vaughan, Encyclopedia of General Topology, Elsevier Science Publishers, B.V., Amsterdam, 2004, x+526 pp.

[3] Work supported by the Caratheodory Programme of the University of Patras.

Universidad de Extremadura, Badajoz
THURSDAY 9, 10:00 – 10:20

Completion of stationary fuzzy metric spaces

Valentín Gregori

Instituto Universitario de Matemática Pura y Aplicada
Universidad Politécnica de Valencia, Spain
e-mail: vgregori@mat.upv.es

Co-author: Almanzor Sapena

AMS *Subject Classification* (2010): 54A40, 54D35, 54E50

Abstract

A fuzzy metric space $(X, M, *)$, in the sense of George and Veeramani, does not admit completion, in general. In this talk we see some aspects relative to the completion of strong fuzzy metrics and then we obtain that a stationary fuzzy metric space $(X, M, *)$ admits completion if $*$ is integral.

Universidad de Extremadura, Badajoz
THURSDAY 9, 10:00 – 10:20

Topological spaces whose topology is generated by a binary relation [4]

Esteban Induráin

Universidad Pública de Navarra, 31006 *Pamplona, Spain*
e-mail: steiner@unavarra.es

Co-author: Vicki Knoblauch [5]

Key words and phrases: binary relations, topological spaces, ordered structures.

AMS *Subject Classification* (2010): 54A10, 54F05

Abstract

We study topologies that are induced by binary relations on sets. First we show that given a binary relation on a nonempty set, a topology is induced in a natural way. Then we consider different classical categories of topological spaces whose topology is defined for at least one binary relation. Given a topology defined by some binary relation on a set, we also analyze if the binary relation could belong to some particular category, with a special attention to different kinds of orderings. We furnish examples of topological spaces whose topology cannot be induced by any binary relation.

References

[1] B. Banaschewski, Recent results in pointfree topology, in " Papers on General Topology and Applications " (Brookville, NY, 1990), 29 – 41, Ann. New York Acad. Sci., 659, New York Acad. Sci., New York, 1992.

[4]This work has been supported by the research project MTM 2007-62499 "Espacios Topológicos Ordenados: Resultados Analíticos y Aplicaciones Multidisciplinares" (Spain).

[5] University of Connecticut, Storrs, CT (U.S.A.).

[2] M.J. Campión, J.C. Candeal, E. Induráin, Preorderable topologies and order-representability of topological spaces, *Topology Appl.* **156** (2009), 2971–2978.

[3] J. Dugundji, "Topology", Allyn and Bacon, Boston, 1966.

[4] R. Engelking, "General Topology. Revised and completed edition", Heldermann Verlag, Berlin, 1989.

[5] P. Erdös, A. Tarski, On families of mutually exclusive sets, *Annals of Math.* **44** (1943), 315–329.

[6] K. Haratomi, Über höherstufige Separabilität unf Kompaktheit I, *Japanese Journal of Math.* **8** (1931), 113–141.

[7] V. Knoblauch, Topologies defined by binary relations, Department of Economics Working Paper Series, Working Paper 2009-28R, University of Connecticut, 2009.

[8] S. Purisch, A history of results on orderability and suborderability, in "Handbook of the History of General Topology", vol. 2, Kluwer, Dordrecht, 1998, 689–702.

[9] L.A. Steen, J.A. Seebach Jr., "Counterexamples in Topology", Dover, New York, 1978.

[10] J. Van Dalen, E. Wattel, A topological characterization of ordered spaces, *General Topology Appl.* **3** (1973), 347–354.

Universidad de Extremadura, Badajoz
FRIDAY 10, 10:40 – 11:00

Permutable quasi-uniformities

HANS-PETER A. KÜNZI

University of Cape Town, Rondebosch 7701, *South Africa*
e-mail: hans-peter.kunzi@uct.ac.za

Co-author: ELIZA P. DE JAGER

AMS *Subject Classification* (2010): 54E15

ABSTRACT

We continue our investigations on the lattice $(q(X), \subseteq)$ of quasi-uniformities on a set X, which we started in [1, 2, 3, 4, 5].

In particular we study quasi-uniformities $\mathcal{U}$ and $\mathcal{V}$ on a set X which permute, that is, $\mathcal{U} \circ \mathcal{V} = \mathcal{V} \circ \mathcal{U}$ (compare [6, 7]).

REFERENCES

[1] E.P. DE JAGER, H.-P. A. KÜNZI, Atoms, anti-atoms and complements in the lattice of quasi-uniformities, *Topology Appl.* **153** (2006), 3140 – 3156.

[2] E.P. DE JAGER, H.-P. A. KÜNZI, Infima of quasi-uniform anti-atoms, *Topology Appl.* **153** (2006), 3327 – 3337.

[3] E.P. DE JAGER, H.-P. A. KÜNZI, Infima and complements in the lattice of quasi-uniformities, *Topology Appl.* **154** (2007), 2117 – 2126.

[4] E.P. DE JAGER, H.-P. A. KÜNZI, The lattice of quasi-uniformities, in "Open Problems in Topology II", edited by E. Pearl, 2007, Elsevier, 691 – 696.

[5] E.P. DE JAGER, H.-P. A. KÜNZI, More about complements of quasi-uniformities, preprint.

[6] W.H. ROWAN, Algebras with a compatible uniformity, *Algebra Univers.* **47** (2002), 13 – 43.

[7] H. WEBER, On lattices of uniformities, *Order* **24** (2007), 249 – 276.

Universidad de Extremadura, Badajoz
THURSDAY 9, 16:50 – 17:10

On G.Λ_π^{gp}-sets, Λ_π^{gp}-closure operator and the associated topology $\tau^{\Lambda_\pi^{gp}}$

Raja M. Latif

Department of Mathematics and Statistics
King Fahd University of Petroleum and Minerals
Dhahran 31261, *Saudi Arabia*
e-mail: raja@kfupm.edu.sa

Co-authors: M. Caldas, S. Jafari, T. Noiri

AMS *Subject Classification* (2000): 54D30, 54A05, 54H05, 54G99

Abstract

In this talk we introduce the concept of Λ_π^{gp}-sets (resp. V_π^{gp}-sets) which is the intersection of πgp-open (resp. union of πgp-closed) sets and investigate the notions of generalized Λ_π^{gp}-sets and generalized V_π^{gp}-sets in a topological space (X, τ). Also we define a new closure operator and thus a new topology $\tau^{\Lambda_\pi^{gp}}$ on (X, τ) by using generalized Λ_π^{gp}-sets and generalized V_π^{gp}-sets and shall examine some of the properties of this new topology.

References

[1] H. Maki, Generalized Λ-sets and the associated closure operator, in "The Special Issue in Commemoration of Prof. Kazuada IKEDA's Retirement", 1986, 139 – 146.

[2] T. Noiri, H. Maki, J. Umehara, Generalized preclosed functions, *Mem. Fac. Sci. Kochi Univ. Ser. A (Math)* **19** (1998), 13 – 20.

[3] Park, On $\pi gp-$closed sets in topological spaces, *Indian J. Pure Appl. Math.* (in press).

[4] J.H. Park , J.K. Park, On $\pi gp-$continuous functions in topological spaces, *Chaos, Solitons and Fractals* **20** (2007), 467 – 477.

[5] M.H. Stone, Applications of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.* **41** (1937), 375 – 481.

[6] V. Zaitsev, On certain classes of topological spaces and their bicompactifications, *Dokl Akad Nauk SSSR* **178** (1968), 778–779.

Universidad de Extremadura, Badajoz
FRIDAY 10, 12:40 – 13:00

On insertion and extension of lattice-valued functions on preordered topological spaces

Iraide Mardones-Pérez

Departamento de Matemáticas, Universidad del País Vasco (UPV-EHU)
Apto. 644, 48080 *Bilbao, Spain*
e-mail: iraide.mardones@ehu.es

Co-authors: Javier Gutiérrez García, María Angeles de Prada Vicente

AMS *Subject Classification* (2010): 54G05, 54D15, 54C20, 54C99, 06D10

Abstract

In this paper, the possibility of inserting an increasing continuous lattice-valued function between two comparable semicontinuous functions is studied. First, we obtain a sufficient condition for such insertion, then we get new characterizations of several classes of preordered topological spaces, among them normally preordered and extremally preorder-disconnected spaces. Conditions for the continuous and increasing extension of lattice-valued maps of the same type defined on closed (resp. open) sets are also investigated.

Universidad de Extremadura, Badajoz
TUESDAY 7, 16:30 – 16:50

[6] On dimension-like functions $dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$ and $Dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$

A.C. Megaritis

University of Patras, Department of Mathematics, 265 04 *Patras, Greece*
e-mail: megariti@master.math.upatras.gr

Co-authors: D.N. Georgiou, S.D. Iliadis

AMS *Subject Classification* (2010): 54B99, 54C25

Abstract

In this paper we give some remarks on the dimension like functions $dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$ and $Dm_{\mathbb{E}}^{\mathbb{K},\mathbb{B}}$ which are defined in [2]. In particular, we give some new dimension-like functions and define using these definitions classes of spaces in which there are universal elements.

References

[1] S.D. Iliadis, "Universal Spaces and Mappings", North-Holland Mathematics Studies, 198, Elsevier Science B.V., Amsterdam, 2005, xvi+559 pp.

[2] D.N. Georgiou, S.D. Iliadis, A.C. Megaritis, Dimension-like functions and universality, *Topology Appl.* **155** (17-18) (2008), 2196 – 2201.

[6] Work supported by the Caratheodory Programme of the University of Patras.

Universidad de Extremadura, Badajoz
TUESDAY 7, 11:30 – 11:50

Recent results on the smallest ideal of $\beta\mathbb{N}$

Gugu Moche

Department of Mathematical Sciences, University of South Africa
P.O.Box 392, 0003, *South Africa*
e-mail: mocheiog@unisa.ac.za

AMS *Subject Classification* (2010): 22A15, 22A20

Abstract

In this presentation we will present a survey of recent results on the structure of the smallest ideal of the Stone-Čech compactification of $\mathbb{N}$.

Universidad de Extremadura, Badajoz
THURSDAY 9, 16:30 – 16:50

On certain covering properties in structured frames

INDERASAN NAIDOO

University of South Africa, P.O. Box 392*, UNISA* 0003*, South Africa*
e-mail: naidoi@unisa.ac.za

AMS *Subject Classification* (2010): 06D22, 18A40, 54D35

ABSTRACT

We consider various covering properties in the category of uniform frames. In particular, we provide characterizations of separability in terms of filters and introduce *pre-Lindelöf* uniform frames. We also look at some of the applications of these covering properties.

Universidad de Extremadura, Badajoz
FRIDAY 10, 16:50 – 17:10

General equilibrium in $C(P)$-type economies

A. Pulgarín

Dept. of Mathematics, Univ. of Extremadura, Cáceres, Spain
e-mail: aapulgar@unex.es

AMS *Subject Classification* (2010): 91B50; 46A40

Abstract

Infinite-dimensional models have become prominent in modern economic theory. Aliprantis–Brown [1] set up as the most appropriate framework to develop an equilibrium theory, an exchange economy consisting in a dual pair $\langle E, P\rangle$ with commodity space E a locally convex-solid vector lattice and price space P its topological dual.

The consumers make rational elections according to some rules allowing them to maximize their benefits by means of certain preferences, and an equilibrium point consists in a feasible allocation cleaning the market for which there is a price system maximizing the preferences among the budget of each consumer. The seminal paper of Mas-Colell [3] provided the first general equilibrium existence theorem by assuming that all the preferences are convex, monotone, continuous and uniformly proper, and therefore represented by utility functions in particular.

Montalvo–Pulgarín–Requejo obtained in [4] an inner characterization of the vector lattice $C(P)$ of real valued continuous functions on a realcompact space P, which endowed with its compact convergence topology becomes a locally convex-solid vector lattice. The first surprised result is that there is a one-to-one correspondence between convex, monotone, continuous and uniformly proper preferences on $C(P)$, quasiconcave lattice seminorms on $C(P)$ and convex compact subsets of P. By considering the nonlinear topological space P as the prices space, we derive the existence of an equilibrium point in $C(P)$ by means of the classical Halpern–Bergman [2] fixed point theorem.

References

[1] C.D. Aliprantis, D.J. Brown, Equilibria in markets with a Riesz space of commodities, *J. Math. Econom.* **11** (1983), 189 – 207.

[2] B.R. Halpern, G.M. Bergman, A fixed point theorem for inward and outward maps, *Trans. AMS* **130** (1968), 353 – 358.

[3] A. Mas-Colell, The price equilibrium existence problem in topological vector lattices, *Econometrica* **54** (1986), 1039 – 1053.

[4] F. Montalvo, A. Pulgarín, B. Requejo, Riesz spaces of continuous functions, *Positivity*, doi: 10.1007/s11117-009-0031-6 (2009).

Universidad de Extremadura, Badajoz
TUESDAY 7, 11:50 – 12:10

Miss topologies compatible with sequential upper Kuratowski-Painlevé convergence

Jesús Rodríguez-López

Instituto Universitario de Matemática Pura y Aplicada
Universidad Politécnica de Valencia, 46022 *Valencia, Spain*
e-mail: jrlopez@mat.upv.es

Co-author: Gerald Beer

AMS *Subject Classification* (2010): 54B20, 54A20, 54E35

Abstract

As it is well-known, upper Kuratowski-Painlevé convergence K^+ for sets is not topological in general. The characterization of when this occurs was given implicitly by Choquet [4] and Dolecki, Greco and Lechicki [6] proving that, for a Hausdorff topological space, the upper Kuratowski-Painlevé convergence is topological if and only if the space is locally compact. In this case, the compatible topology coincides with the cocompact topology $\mathcal{T}_C$. In [6], it is introduced the concept of a *consonant space*, which is a space where the topological modification of the Kuratowski-Painlevé convergence coincides with the cocompact topology (see also, for example, [1, 2, 8]).

Later on, Costantini, Holá and Vitolo [5] studied the notion of *sequential consonance*. A topological space is said to be sequentially consonant if K^+ and $\mathcal{T}_C$ have the same convergent sequences to the same points, i. e. these convergences are *sequentially equivalent*. In [5, 7] it is proved that every k-space or P-space is sequentially consonant so, in particular, every locally compact space or every first countable space so is.

In [3], we have studied a question related to the above problem. Concretely, we try to characterize the miss topologies which are sequentially equivalent to the upper Kuratowski-Painlevé convergence. Furthermore, we look more carefully at such miss topologies when the underlying topology of the space is first countable.

References

[1] B. Alleche, J. Calbrix, On the coincidence of the upper Kuratowski topology with the cocompact topology, *Topology Appl.* **93** (3) (1999), 207 – 218.

[2] M. Arab, J. Calbrix, Sur la topologie de Fell et la topologie de la convergence, *C. R. Acad. Sci., Paris, Sér. I* **318** (1994), 549 – 552.

[3] G. Beer, J. Rodríguez-López, Topologies associated with Kuratowski-Painlevé convergence of closed sets, *J. Convex Anal.* **17** (3-4) (2010), to appear.

[4] G. Choquet, Convergences, *Ann. Univ. Grenoble. Sect. Sci. Math. Phys.* **23** (1947-1948), 57 – 112.

[5] C. Costantini, L. Holá, P. Vitolo, Tightness, character and related properties of hyperspace topologies, *Topology Appl.* **142** (1-3) (2004), 245 – 292.

[6] S. Dolecki, G. Greco, A. Lechicki, When do the upper Kuratowski topology (homeomorphically scott topology) and the co-compact topology coincide?, *Trans. Amer. Math. Soc.* **347** (8) (1995), 2868 – 2883.

[7] J.-C. Hou, A note on the Fell topology, *Quest. Answers Gen. Topology* **17** (1999), 81 – 87.

[8] T. Nogura, D. Shakmatov, When does the Fell topology on a hyperspace of closed sets coincide with the meet of the upper Kuratowski and the lower Vietoris topologies?, *Topology Appl.* **70** (2-3) (1996), 213 – 243.

Universidad de Extremadura, Badajoz
THURSDAY 9, 11:30 – 11:50

Fractal dimensions for fractal structures: a Haussdorf dimension generalization

M.A. Sánchez-Granero

Área de Geometría y Topología, Universidad de Almería (UAL)
04071 *Almería, Spain*
e-mail: misanche@ual.es

Co-author: M. Fernández-Martínez

AMS *Subject Classification* (2010): 28A80, 54E15, 54E35

Abstract

A fractal structure is a sequence of coverings (or levels) of a space with some properties. They were introduced to charaterize nonarchimedean quasi-metrizability and used to study self similar sets.

In this talk we present some definitions of fractal dimensions for a fractal structure and compare the relation between them and the classical box-counting and Hausdorff dimensions.

It is clear from these definitions that fractal structures can be used to provide a good model to generalize the concepts of box counting and Hausdorff dimensions.

In particular, we will focus on one of this definitions and prove that it coincides with the Hausdorff dimension for the natural fractal structure on an euclidean space, which is given by tilings of cubes of side $\frac{1}{2^n}$.

Universidad de Extremadura, Badajoz
FRIDAY 10, 10:20 – 10:40

Metrics and fuzzy metrics

Almanzor Sapena

Instituto Universitario de Matemática Pura y Aplicada
Universidad Politécnica de Valencia, Spain
e-mail: alsapie@mat.upv.es

Co-author: Valentín Gregori

AMS *Subject Classification* (2010): 54A40, 54D35, 54E50

Abstract

In this talk we study strong fuzzy metric spaces and we find for a strong and principal fuzzy metric space $(X, M, *)$ a family of stationary fuzzy metrics which are compatible with the topology τ_M induced by M. From the last result, if $* \geq \mathfrak{L}$, where $\mathfrak{L}$ is the Lukasievicz t-norm, we construct a family of classical metrics that induce the same topology τ_M on X. We illustrate the results by appropriate examples.

Universidad de Extremadura, Badajoz
TUESDAY 7, 12:10 – 12:30

Pseudoradial order of pseudoradial spaces

Gino Tironi [7]

Dipartimento di Matematica e Informatica, Università di Trieste
Via Alfonso Valerio 12/1, 34127 *Trieste, Italy*
e-mail: tironi@units.it

Co-author: Stefano Modena [8]

Key words and phrases: pseudoradial spaces, pseudoradial order, adjunction spaces.

AMS *Subject Classification* (2010): 54A20, 54A25, 54B17

Abstract

Pseudoradial normal spaces of any order of pseudoradiality given by an ordinal number not greater than $\sigma_c(X)^+$ are constructed. Another construction with similar properties is given for compact T_0 spaces. Finally pseudoradial spaces of cardinality ω_α and pseudoradial order $\omega_{\alpha+1}$ are exhibited. The most important tool to perform such constructions is the pseudoradial sum and the evaluation of its order of pseudoradiality.

References

[1] A.V. Arhangel'skiĭ, The structure and classification of topological spaces and cardinal invariants, *Uspekhi Mat. Nauk* **33** (1978), 29 – 84.

[2] A.V. Arhangel'skiĭ, S.P. Franklin, Ordinal invariants for topological spaces, *Michigan Math. J.* **15** (3) (1968), 313 – 320.

[3] A.V. Arhangel'skiĭ, R. Isler, G. Tironi, On pseudo-radial spaces, *Comment. Math. Univ. Carolinae* **27** (1986), 137 – 156.

[7] Research supported by University of Trieste within the Program: Finanziamento Ricerca d'Ateneo.

[8] Co-author e-mail: *stefano.modena@hotmail.it*

[4] A. Bella, G. Tironi, d-4 Pseudoradial spaces, in "Encyclopedia of General Topology", Elsevier, 2003, 165 – 168.

[5] M. Dolcher, Topologie e strutture di convergenza, *Ann. Scuola Norm. Sup. Pisa* **14** (1960), 63 – 92.

[6] H. Herrlich, Quotienten geordneter Räume und Folgenkonvergenz, *Fund. Math.* **61** (1967), 79 – 81.

[7] R. Vaidyanathaswamy, "Set Topology", second edition, Chelsea Publishing Company, New York, 1960.

Universidad de Extremadura, Badajoz
FRIDAY 10, 16:30 – 16:50

The Stone-Čech compactification of topological groups

Tero Vedenjuoksu

University of Oulu, Finland
e-mail: tero.vedenjuoksu@oulu.fi

Co-author: Mahmoud Filali [9]

AMS *Subject Classification* (2010): 22A05, 54H11

Abstract

We consider the problem when the Stone-Čech compactification βG of a topological group G is a semigroup with an operation extending that of G such that G is contained in the topological centre of βG. In this case, we shall say that G has the β-extension property. We shall use so-called left (resp. right) multiplicatively continuous complex-valued functions on G.

A topological group for which countable intersections of open sets remains open is called a P-group. We shall show that a topological group G, which is not a P-group, has the β-extension property only when G is pseudocompact. In general, P-groups does not have the β-extension property. We shall show that Lindelöf P-groups have the β-extension property. We shall consider some examples of P-groups having the β-extension property as well as not having the β-extension property, and discuss some further problems related to the subject.

References

[1] M. Filali, T. Vedenjuoksu, The Stone-Čech compactification of topological groups and the β-extension property, *Houston J. Math.* **36** (2) (2010), 477–488.

[9] University of Oulu, Finland

Participants

Badajoz (Spain)

September 7 – 10, 2010

Participants :

Fulya Akiz, *Bozok University, Turkey*

Teresa Arias-Marco, *Universidad de Extremadura, Spain*

Mohamed Bakier, *Assiut University, Egypt*

Francisco Balibrea, *Universidad de Murcia, Spain*

Angelo Bella, *Università di Catania, Italy*

Gianni Bosi, *University of Trieste, Italy*

Bruce Burdick Roger, *Williams University, USA*

Félix Cabello, *Universidad de Extremadura, Spain*

Joan Gerard Camarena Estruch, *Universidad Politécnica de Valencia, Spain*

Eduardo Corbacho Cortés, *Universidad de Extremadura, Spain*

María Angeles De Prada Vicente, *Universidad del País Vasco-Euskal Herriko Unibertsitatea, Spain*

Anna Di Concilio, *University of Salerno, Italy*

Dikran Dikranjan, *Udine University, Udine, Italy*

Jesús M. Domínguez, *Universidad de Valladolid, Spain*

Erdal Ekici, *Canakkale Onsekiz Mart University, Turkey*

Manuel Fernández Martínez, *Universidad de Almería, Spain*

Maria João Ferreira, *University of Coimbra, Portugal*

María Vicenta Ferrer, *Universitat Jaume I, Spain*

Ricardo García, *Universidad de Extremadura, Spain*

Juan-Luis García Zapata, *Universidad de Extremadura, Spain*

M. Isabel Garrido, *Universidad Complutense de Madrid, Spain*

M. Teresa Gassó Matoses, *Universidad Politécnica de Valencia, Spain*

Dimitrios Georgiou, *University of Patras, Greece*

Adrián Gordillo, *Universidad de Extremadura, Spain*

Valentín Gregori, *Universidad Politécnica de Valencia, Spain*

Salvador Hernández Muñoz, *Universitat Jaume I, Spain*

Miroslav Husek, *Charles University, University J.E. Purkyne, Czech Republic*

Murad Hussain, *COMSATS Institute of Information Technology, Islamabad, Pakistan*

Esteban Induráin, *Universidad Pública de Navarra, Spain*

Jesús A. Jaramillo, *Universidad Complutense de Madrid, Spain*

Gulen Karakoc, *Bilkent University, Turkey*

Hans-Peter Kunzi, *University of Cape Town, South Africa*

Raja Mohammad Latif, *King fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia*

Carlos Lorenzo, *Universidad de Valladolid, Spain*

Iraide Mardones, *Universidad del País Vasco-Euskal Herriko Unibertsitatea, Spain*

Athanasios Megaritis, *University of Patras, Greece*

Gugu Moche, *University of South Africa, South Africa*

Francisco Montalvo, *Universidad de Extremadura, Spain*

Roberto Moresco, *Università di Padova, Italy*

M. Ángeles Mulero, *Universidad de Extremadura, Spain*

Macarena Muñoz, *Universidad de Extremadura, Spain*

Inderasan Naidoo, *University of South Africa, South Africa*

Jorge Picado, *University of Coimbra, Portugal*

Antonio A. Pulgarín, *Universidad de Extremadura, Spain*

Batildo Requejo, *Universidad de Extremadura, Spain*

Rosario de Gracia Rodríguez, *Universidad de Extremadura, Spain*

Jesús Rodríguez-López, *Universidad Politecnica de Valencia, Spain*

Miguel Angel Sánchez-Granero, *University of Almería, Spain*

Manuel Sanchis, *Universitat Jaume I, Spain*

Almanzor Sapena, *Universidad Politécnica de Valencia, Spain*

Gino Tironi, *Università di Trieste, Italy*

Oscar Valero, *Universidad de las Islas Baleares, Spain*

Tero Vedenjuoksu, *University of Oulu, Finland*